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ABSTRACT

The study investigated whether 15 children (ages 6-21) with IQs from 31 to 66 could spontaneously invent more efficient calculational procedures and abstract basic arithmetic relationships after individualized tutoring in computation. Experimental subjects were given training that focused on accurate computation but specific relationships and calculational shortcuts were not pointed out. As a result of the computational training, many experimental subjects spontaneously invented shortcuts for the concrete counting-all procedure they already knew including the "counting fingers strategy." More experimental than control children (who received training in nonarithmetical areas) discovered that applying the commutativity principle could shortcut their computational effort, and more experimental than control subjects mastered facts involving zero and one. (DB)

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Addition Learning by Mentally Handicapped Children

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Abstract

A training experiment was undertaken with 30 children who ranged in IQ from 31 to 66 to see if mentally handicapped children could spontaneously invent more efficient calculational procedures and abstract basic arithmetic relationships. Some experimental subjects did invent calculational shortcuts. The most common was what Siegler and Robinson (1982) call the "counting-fingers strategy": represent each addend with a finger pattern and then count all the fingers put up. Some even adopted what these researchers call a "fingers strategy": represent each addend with a finger pattern and, without counting, announce the total number of fingers put up. From their computational training, the experimental subjects apparently discovered the commutativity principle: The order of the addends does not affect the outcome. When presented commuted problems in sequence (e.g., when 3 + 5 followed 5 + 3), significantly more experimental subjects than control subjects shortcut their computational effort. That is, they did not bother to compute the sum of the second problem but simply stated the previously calculated sum. The experimental group also demonstrated significantly greater mastery of combinations involving zero and one, including those not practiced. Experimental subjects apparently induced a zero rule (adding zero leaves a number unchanged) and a rule for adding one: The sum is the number after the other term (e.g., 8 follows 7 when we count, so 1 + 7 is 8). Even mentally handicapped children can master the addition combinations involving zero and one by discovering and exploiting relationships rather than by practicing each combination separately (cf. Ashcraft, 1985; Siegler & Shrager, 1984).

Aims

Research (e.g., Baroody, 1987b; Carpenter & Moser, 1984; Groen & Resnick, 1977; Ilg & Ames, 1951) indicates that preschool and primary-level children spontaneously invent more efficient procedures for calculating the sums of single-digit addition problems (Resnick, 1983; Resnick & Ford, 1981). There is some evidence (e.g., Baroody & Gannon, 1984; Baroody, Ginsburg, & Waxman, 1983) that--without direct instruction--primary-level children learn basic arithmetic relationships, such as the commutativity principle (the order in which two addends are combined does not affect the outcome or sum). There is some data (Baroody, 1985; Olander, 1931; Thiele, 1938) that suggest children abstract and exploit relationships in mastering combinations involving zero (adding zero does not affect a number) and one (when adding one the sum is the number after the other addend in the count sequence). By exploiting relationships, such as the commutativity principle and the zero and one rules, children do not have to practice each single-digit combination in order to achieve mastery or efficient recall.

The aim of this training experiment was to see if children classified as mentally handicapped are capable of similar types of learning. Specifically, can such children:

(a) invent more economical counting procedures for adding;

(b) abstract the commutativity principle; and

(c) induce zero and one rules, which would transfer to unpracticed combinations.

Method & Subjects

Subjects were drawn from 11 classes in an upstate New York county-wide special education service agency. Screening for prearithmetic skills identified 30 children that were suitable for the study. The sample consisted of 24 children classified as moderately mentally handicapped (IQs ranging between 31 and 49) and 6 children classified as mildly mentally handicapped (IQs ranging from 52 to 66). Children ranged in chronological age from 6 years and 10 months to 20 years and 10 months. There were an equal number of males and females.

The sample was individually pretested on a calculational, commutativity, and mental-addition task. The children were randomly assigned to a control group and an experimental group. All children were individually tutored by one of three graduate assistants for a period of 20 weeks. There was a total of 51 20-minute sessions ($\underline{M}$ = 2.55 sessions per week). The control group was tutored on IEP mathematics objectives not related to arithmetic (e.g., identifying the value of coins, telling time to the half hour). The experimental group was given training that focused on helping them to compute accurately. The trainers did not point out specific relationships, such as the zero rule. Except for 1 + 6, 2 + 6, and 6 + 2, the experimental training involved problems with addends of 0 to 5 only.

The children were retested by testers who were blind to subject assignments. The mental-addition posttest included combinations that were not practiced during the training phase as well as those that were.

Tasks

Screening for prearithmetic skills. To qualify for the study, a child had to demonstrate competence in comparing the magnitude of numbers one to five, reading numerals to 10, producing sets of one to five objects, and enumerating 1 to 12 objects.

Computational task. This task consisted of randomly presenting a set of addition combinations to a child on two occasions. The first set consisted of 1 + 3, 1 + 9, 2 + 1, 2 + 4, 3 + 2, 3 + 6, 4 + 5, 5 + 3, 7 + 1, and 8 + 2; the second set consisted of the commuted counterparts of the combinations listed above. The child was instructed to figure out the answer any way he or she wanted--with blocks, fingers, or in his or her head. The solution strategy was scored using the criterion specified in Baroody (1987b). Accuracy was also noted.

Commutativity task. A sequence of addition sentences was presented. Half the time, a combination was followed by a commuted trial; the other half, by a noncommuted problem with a different sum. A child was scored as successful on the task if he or she did not compute the sum of the commuted combination but simply recorded the previously calculated sum. (Noncommuted trials were included to deter or detect a response bias).

Mental-addition task. The mental-addition task entailed randomly administering a set of 16 combinations 20 times in 7 or 8 sessions over the course of 4 weeks. A combination was presented as a written number sentence (e.g., 5 + 3) and verbally (e.g., "This says five and three. Quickly, how much is five and three altogether?"). The subjects were required to answer without computing. A time limit minimized the possibility that children mentally reconstructed sums by using a thinking strategy or by computing. The testing was audiotaped.

The test items included combinations of three different types: those involving zero, one, and numbers greater than one. Half the items were ascending (smaller-addend-first) combinations, the other half were descending (smaller-addend-last) combinations. Items (0 + 5, 0 + 9, 4 + 0, 6 + 0, 1 + 4, 1 + 7, 3 + 1, 8 + 1, 2 + 5, 3 + 4, 5 + 8, 7 + 9, 4 + 2, 5 + 3, 8 + 6, and 9 + 3) were presented in random order.

Three criteria defined mastery of combinations involving zero or one. First, correct posttest responses for all four test items for a given type of problem had to exceed 85%. Mastery then entailed substantial success on two problems not practiced in the experimental condition as well as two that were practiced. Second, a subject had to respond discriminately to zero and one problems. Combinations involving zero were not considered mastered if the subject's predominant response to both combinations involving zero and one was the larger addend. Combinations involving one were not considered mastered if the subject's predominant response to both combinations involving one and zero was the larger addend plus one. Third, to discount random fluctuations in performance, the subject had to demonstrate a gain of 5 percentage points or more on the posttest.

Three-addend task. This task was administered on the posttest only. The procedures described for the mental-addition task were followed. The three-addend items were 0 + 0 + 2, 0 + 0 + 5, 3 + 0 + 0, 4 + 0 + 0, 1 + 1 + 4, 2 + 1 + 1, and 5 + 1 + 1.

Experimental Training

The experimental training consisted of four phases. In Phase 1, each addend of the addition problems was represented by dots within a 7.62 cm x 7.62 cm box on a 12.8 cm x 20.4 cm card (see Figure 1). The dots were arranged in a regular pattern as on a die. An empty box represented zero. Below each box the cardinal value of the addend was indicated by a numeral. A plus sign was positioned between the two numerals. A total of 36 cards was used to play a variety of math games (see Baroody, 1987, in press-a, for details). If the child did not respond or used his or her own strategy to generate an incorrect answer, the trainer had the child count the two sets of dots.

In Phases 2 to 4, the problems were represented on 7.7 cm x 12.8 cm cards using numerals only. In Phase 2, blocks were provided and, if needed, the child was instructed or helped to use a concrete counting-all procedure. In Phase 3, an abacus-like device with five red markers on one side and five green markers on other was provided. If needed, the child could compute the sum of a problem by sliding up the appropriate number of markers to represent each addend and then counting the number of markers of both colors in the up position. In Phase 4, nonresponders or incorrect responders were encouraged or helped to use their fingers to compute the sums of problems. Initially, the four phases each lasted 2 weeks; thereafter each phase was repeated in turn for an interval of 1 week.

Results

Computational Shortcuts. As a result of their computational training, many experimental subjects spontaneously invented shortcuts for the concrete counting-all procedure they were taught or already knew. The most common shortcut was what Siegler and Robinson (1982) call the "counting-fingers strategy": automatically representing each addend with finger patterns (rather than counting out objects) and then counting all the fingers displayed. Some children even adopted what Siegler and Robinson call the "fingers strategy": automatically represent each addend on fingers of each hand and then, without further counting, announce the total (sum). Though the experimental subjects learned concrete procedures for computing sums, none invented a mental algorithm. This parallels results (Baroody, 1987b) that indicate that kindergartners persist in their use of concrete computing procedures but find many ways to shortcut them.

Commutativity. During the training phase, many experimental subjects discovered that when commuted problems were presented successively, they could shortcut their computational effort by using previously computed sums. On the posttest, the experimental children significantly ($p < .05$, one-tailed, Fisher Exact 2 x 2 Test) outperformed those in control group on a simple problem-solving task that required the application of commutativity. When presented commuted pairs of problems in sequence (e.g., when 3 + 5 followed 5 + 3 instead of 3 + 1), experimental children did not bother to compute the sum of the second problem but simply stated the previously calculated sum.

Mastery of facts involving zero and one. Six of 10 experimental subjects mastered the combinations involving zero between the pre- and posttests (mean gain for those who achieved criterion was +56% correct; for

the experimental group as a whole, +30% correct). Only 1 of 12 controls achieved mastery (mean gain for this group was -5%). The difference is statistically significant (p = .012, one-tailed, Fisher Exact 2 x 2 Test). Thus, factors like maturation can be discounted (cf. Ashcraft, 1985). Furthermore, five experimental subjects achieved criterion on the three-addend task (p = .09, one-tailed, Fisher Exact 2 x 2 Test).

Four of 11 experimental children achieved criterion on the combinations involving one between the testing phases (mean gain for those achieving criterion was +47% correct; for the experimental group as a whole, +36%). None of the 14 control children who initially did not know such combinations did so (mean gain for the control group was +8%). The difference between the groups was statistically significant (p = .03, one-tailed, Fisher Exact 2 x 2).

Conclusions

Without explicit instruction, some experimental children did invent shortcuts for the concrete calculational procedure they were taught or already knew. Apparently, even children who are classified as mentally handicapped do monitor and spontaneously adjust their mathematical behavior (Baroody, 1987a). Helping such children make the relatively difficult transition from computing with objects to counting mentally may require more lengthy or direct instruction.

Without direct instruction, the experimental children apparently discovered that the order of the addends does not affect the outcome. These results are consistent with other data (e.g., Baroody & Snyder, 1983) that suggests that children classified as mentally handicapped abstract simple mathematical regularities.

The qualitative change in the experimental subjects' responses on the mental-addition task suggests that they learned a general rule for adding zero and one. These results challenge the view (cf., Ashcraft, 1985; Siegler & Shrager, 1984) that such combinations are internalized and stored individually (Baroody & Ginsburg, 1986). Even children classified as mentally handicapped can master addition combinations involving zero and one by discovering and exploiting relationships (rather than practicing each separately). In the case of adding one, a child need only to see the connection between these combinations and their existing knowledge of number successors (Baroody, 1987, in press-b). Once children grasp this (number-after) relationship (see Fuson, Richards, & Briars, 1982), they can solve efficiently any problem with one as an addend for which they know the other number's successor--including those not previously practiced (cf. Thorndike, 1922).

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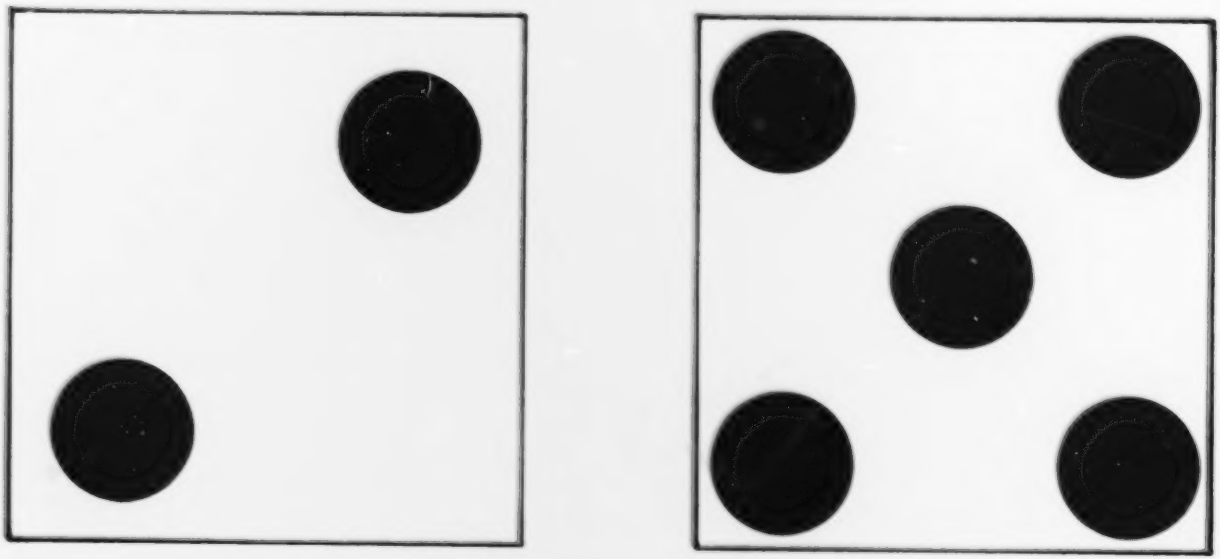
Figure 1

Example of the Stimulus

Used in Phase 1 of the

Experimental Training



2 + 5